ABSTRACT. The problem of generalized thermoelasticity of two-temperature for finite piezoelectric rod will be modified by applying three different types of heating applications namely, thermal shock, ramp-type heating and harmonically vary heating. The solutions will be derived with direct approach by the application of Laplace transform and the Caputo-Fabrizio fractional order derivative. The inverse Laplace transforms are numerically evaluated with the help of a method formulated on Fourier series expansion. The results obtained for the conductive temperature, the dynamical temperature, the displacement, the stress and the strain distributions have represented graphically using MATLAB.

1. INTRODUCTION

Many theories have been discovered and tried to eliminate the contradiction of the infinite speed of heat propagation in the classical theory of thermoelasticity. But two among them are very famous. That are Lord-Shulman theory [1] and Green-Lindsay theory [2]. Many of the authors gave their contribution in the theory of generalized Thermopiezoelectricity for two-temperature as seen in the work of Youssef et.al. [3], Tianhu et.al. [4], Nowacki [5], Abouelregal [6] and Chandrasekharaihaiah [7].

In last few decades there had been remarkable developments in the field of Fractional Calculus as shown in the many researchers work like Podlubny [8], Miller et.al. [9], Yang et.al. [10] and Losada et.al. [11]. The use of derivative of fractional order has also spread into the work of many researchers as Raslan [12], Sherief et.al. [13, 14, 15], Youssef et.al. [16, 17], and Ezzat et.al. [18]. Honig et.al. [19], discussed the numerical inversion of Laplace transform. Some contributions of this theory are in[20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

This paper is the modification of the work of E. Bassiouny and H. M. Youssef [35]. In this paper, a new thermoelastic model has been prepared with Caputo-Fabrizio fractional derivative.
of order $0 \leq \alpha \leq 1$ and the results are derived in Laplace transform domain defined in [36] and using the direct approach. The comparative study of Thermopiezoelectricity of finite rod with three different types of thermal loading has been done and the conductive temperature, the dynamical temperature, the stress, the strain and the displacement functions are to be determined. Numerical results are illustrated graphically using MATLAB software.

**Nomenclature:**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda, \mu$</td>
<td>Lame’s Constants</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Two-temperature parameter</td>
</tr>
<tr>
<td>$C_E$</td>
<td>Specific heat at constant strain</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Reference temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Ramping time parameter</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Components of displacement vector</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>The angular frequency of thermal vibration</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta function</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>Coefficient of linear thermal expansion</td>
</tr>
<tr>
<td>$\xi$</td>
<td>The entropy density</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The conductive temperature</td>
</tr>
<tr>
<td>$q_i$</td>
<td>The components of the heat flux vector</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>Strain tensor component</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>Stress tensor component</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>One relaxation time parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fractional order operator</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$(3\lambda + 2\mu)\alpha_T$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$ac_0^2/T_0^2$, Dimensionless two-temperature parameter</td>
</tr>
<tr>
<td>$c$</td>
<td>$\rho C_E/T_0$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$(T - T_0)$, The dynamical temperature increment such that, $</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\rho C_E/k$, The thermal viscosity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$(\lambda + 2\mu)/T_0$, Dimensionless thermoelastic coupling constant</td>
</tr>
<tr>
<td>$c_0$</td>
<td>$\sqrt{\lambda + 2\mu}/\rho$, Longitudinal wave speed</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$\rho C_E$, Dimensionless mechanical coupling constant</td>
</tr>
<tr>
<td>$D$</td>
<td>Constant, the component of electric displacement</td>
</tr>
<tr>
<td>$F_0$</td>
<td>Constant, the strength of shock on the boundary</td>
</tr>
</tbody>
</table>
2. **One-dimensional formulation of the problem**

Consider a piezoelectric rod of finite length $h$. At initial stage the rod is at rest. The one end of the rod is being heated and other is fixed at initial temperature. Here, the displacement component is depends only on $x$ co-ordinate. Thus, we have,

$$u_x = u(x, t), \quad u_y = u_z = 0$$

The initial conditions are assumed as follows:

$$u(x, 0) = 0, \quad 0 \leq x \leq h$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 \leq x \leq h$$

$$\sigma(x, 0) = 0, \quad 0 \leq x \leq h$$

$$\phi(x, 0) = \dot{\phi}(x, 0) = 0, \quad 0 \leq x \leq h$$

Suppose, $F(t)$ is a heating function which is being applied to one end of the rod and as the rod is traction free, the boundary conditions are as follows:

$$\phi_0(0, t) = F(t) \quad (2.1)$$

$$e(0, t) = e(h, t) = 0, \quad \sigma(0, t) = 0 \quad (2.2)$$

For this problem we will consider the following basic equations:

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \theta}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (2.3)$$

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \theta - hD \quad (2.4)$$

$$k \frac{\partial^2 \phi}{\partial x^2} = \left( \frac{\partial}{\partial \xi} + \tau \frac{\partial^{\alpha+1}}{\partial \xi^{\alpha+1}} \right) \left[ \rho C_E \theta + \gamma T_0 e \right] \quad (2.5)$$

$$\phi - T = a \frac{\partial^2 \phi}{\partial x^2} \quad (2.6)$$

$$e = \frac{\partial u}{\partial x} \quad (2.7)$$

$$E = -\frac{\partial v}{\partial x} \quad (2.8)$$

$$\frac{\partial D}{\partial x} = 0 \quad (2.9)$$

For our convenience, we consider the following non-dimensional variables,

$$u' = c_0 \eta u, \quad \phi' = \frac{\phi - T_0}{T_0}, \quad \tau' = \xi' \tau, \quad \sigma' = \frac{\sigma}{\lambda + 2\mu}, \quad t' = c_0 \eta t, \quad \theta' = \frac{T - T_0}{T_0}.$$
Thus, the equations from Eq.(2.3)–Eq.(2.9)(dropping the primes) takes the form,
\[
\frac{\partial^2 e}{\partial x^2} - \beta \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} \tag{2.10}
\]
\[
\sigma = e - \beta \theta - D \tag{2.11}
\]
\[
\frac{\partial^2 \phi}{\partial x^2} = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}} \right) (\theta + e) \tag{2.12}
\]
\[
\theta = \phi - \omega \frac{\partial^2 \phi}{\partial x^2} \tag{2.13}
\]

3. **Solution in the Laplace Transform Domain**

The Laplace transform defined as follows:
\[
L[f(t)] = \tilde{f}(s) = \int_0^\infty e^{-st} f(t) dt \tag{3.1}
\]

The Caputo-Fabrizio fractional order derivative is defined in [36] as follows:
\[
a_D^{(\alpha)} t f(t) = \frac{M(\alpha)}{1-\alpha} \int_a^t f'(\tau) \exp \left[ -\frac{\alpha(t-\tau)}{1-\alpha} \right] d\tau \tag{3.2}
\]
where, \(M(\alpha)\) is the normalization function such that,
\[
M(0) = M(1) = 0, \quad 0 \leq \alpha \leq 1, \quad -\infty < a < t, \quad f \in H^1(a,b), \quad a < b
\]

We suppose that the function \(M(\alpha) = 1\) and substituting \(a = 0\) in the definition defined in Eq.(3.2), we obtain the Laplace transform of Caputo-Fabrizio fractional derivative in \(s\) variable as follows:
\[
L[0_D^{(\alpha)} t f(t)] = \frac{1}{1-\alpha} \int_0^\infty e^{-st} f'(\tau) \exp \left[ -\frac{\alpha(t-\tau)}{1-\alpha} \right] d\tau dt \tag{3.3}
\]
this implies,
\[
L[0_D^{(\alpha)} t f(t)](s) = \frac{s L[f(t)] - f(0)}{s + \alpha(1-s)}, \quad 0 \leq \alpha \leq 1
\]

Similarly,
\[
L[0_D^{(\alpha+1)} t f(t)](s) = \frac{s^2 L[f(t)] - sf(0) - f'(0)}{s + \alpha(1-s)}, \quad 0 \leq \alpha \leq 1 \tag{3.3}
\]

Taking Laplace transform of Eqs.(2.10)–(2.13) using the Eqs.(3.1) and (3.3), we get,
\[
\frac{d^2 \tilde{e}}{dx^2} - \beta \frac{d^2 \tilde{\theta}}{dx^2} = s^2 \tilde{e} \tag{3.4}
\]
\[
\bar{\sigma} = \bar{\epsilon} - \beta \bar{\theta} - \frac{D}{s}
\]  
(3.5)

\[
\frac{d^2 \bar{\phi}}{dx^2} = \left[ s + \frac{\tau_0 s^2}{s + \alpha(1 - s)} \right] \bar{\theta} + \epsilon \left[ s + \frac{\tau_0 s^2}{s + \alpha(1 - s)} \right] \bar{\epsilon}
\]  
(3.6)

\[
\bar{\theta} = \bar{\phi} - \omega \frac{d^2 \bar{\phi}}{dx^2}
\]  
(3.7)

The boundary conditions (2.1)–(2.2) takes the form,

\[
\bar{\phi}(0, s) = \bar{F}(s), \quad \bar{\phi}(h, s) = 0
\]  
(3.8)

\[
\bar{\epsilon}(0, s) = \bar{\epsilon}(h, s) = 0, \quad \bar{\sigma}(0, s) = 0
\]  
(3.9)

Now by combining Eqs.(3.6)–(3.7), we obtain,

\[
\frac{d^2 \bar{\phi}}{dx^2} = L \bar{\phi} + L \epsilon \bar{\bar{\epsilon}}
\]  
(3.10)

where,

\[
L = \frac{(1 + \tau_0)s^2 + \alpha s(1 - s)}{(1 + \omega s)(s + \alpha(1 - s)) + \omega \tau_0 s^2}
\]

Putting the value from Eq.(3.10) into Eq.(3.7), we obtain,

\[
\bar{\theta} = (1 - \omega L)\bar{\phi} - \omega L \epsilon \bar{\bar{\epsilon}}
\]  
(3.11)

Differentiating Eq.(3.11) twice w.r.t. \(x\), we obtain,

\[
\frac{d^2 \bar{\theta}}{dx^2} = (1 - \omega L)\frac{d^2 \bar{\phi}}{dx^2} - \omega L \epsilon \frac{d^2 \bar{\epsilon}}{dx^2}
\]  
(3.12)

Substituting this value from Eq.(3.12) into Eq.(3.4) and using Eq.(3.10), we obtain,

\[
\frac{d^2 \bar{\epsilon}}{dx^2} = M \bar{\phi} + N \bar{\epsilon}
\]  
(3.13)

where,

\[
M = \frac{\beta L(1 - \omega L)}{1 + \beta \omega L \epsilon}, \quad N = \frac{s^2 + \beta L \epsilon(1 - \omega L)}{1 + \beta \omega L \epsilon}
\]
4. Direct Approach

Eliminating $\bar{\phi}$ between Eqs.(3.10) and (3.13), we get,
\[ \left[ \frac{d^4}{dx^4} - (L + N) \frac{d^2}{dx^2} + (LN - \epsilon LM) \right] \bar{\phi} = 0 \]  
(4.1)

The solution of Eq.(4.1) can be written as follows:
\[ \bar{\phi}(x,s) = \sum_{k=1}^{2} (A_k e^{-xP_k} + B_k e^{xP_k}) \]  
(4.2)

Similarly, eliminating $\bar{e}$ between Eqs.(3.10) and (3.13), we get,
\[ \left[ \frac{d^4}{dx^4} - (L + N) \frac{d^2}{dx^2} + (LN - \epsilon LM) \right] \bar{e} = 0 \]  
(4.3)

and the solution of Eq.(4.3) takes the form as,
\[ \bar{e}(x,s) = \sum_{k=1}^{2} (C_k e^{-xP_k} + D_k e^{xP_k}) \]  
(4.4)

where, $A_k, B_k, C_k$ and $D_k$, for $k = 1, 2$ are all parameters depending on $s$.

By applying boundary conditions defined in Eq.(3.8) and Eq.(3.9) on Eqs.(4.2) and (4.4), we can determined the parameters as follows:

\[ A_1 = \frac{\phi_0(L - P_1^2)e^{2hP_1}}{2(P_1^2 - P_2^2)\sin h(hP_1)}, \quad B_1 = \frac{\phi_0(P_1^2 - L)}{2(P_1^2 - P_2^2)\sin h(hP_1)} \]  
(4.5)

\[ A_2 = \frac{\phi_0(P_2^2 - L)e^{2hP_2}}{2(P_1^2 - P_2^2)\sin h(hP_2)}, \quad B_2 = \frac{\phi_0(L - P_2^2)}{2(P_1^2 - P_2^2)\sin h(hP_2)} \]  
(4.6)

from these relations we can write,

\[ C_k = \frac{(P_2^2 - L)A_k}{\epsilon L}, \quad D_k = \frac{(P_2^2 - L)B_k}{\epsilon L}, \quad k = 1, 2 \]  
(4.7)

Here $\pm P_1, \pm P_2$ are zeros of characteristic equation,
\[ P^4 - (L + N)P^2 + LN - \epsilon LM = 0 \]

and which satisfy the relations,
\[ P_1 + P_2 = L + N \]
\[ P_1 \cdot P_2 = LN - \epsilon LM \]

Substituting the values from Eqs.(4.5)–(4.7) in Eq.(4.2) and Eq.(4.4), we obtain the conductive heat function and strain function as follows:

\[ \phi(x,s) = \phi_1 \sin h[(h - x)P_1] + \phi_2 \sin h[(h - x)P_2] \]  
(4.8)

where
\[ \phi_1 = 2A_1 e^{-2hP_1} \quad \text{and} \quad \phi_2 = 2A_2 e^{-2hP_2} \]  
(4.9)
and
\[ \bar{e}(x, s) = e_1 \sin h[(h - x)P_1] + e_2 \sin h[(h - x)P_2] \quad (4.10) \]
where
\[ e_1 = 2C_1 e^{-2hP_1} \quad \text{and} \quad e_2 = 2C_2 e^{-2hP_2} \quad (4.11) \]
Substituting Eqs.(4.8) and (4.10) in Eq.(3.11), we obtain the thermodynamical heat function as follows:
\[ \bar{\theta}(x, s) = \theta_1 \sin h[(h - x)P_1] + \theta_2 \sin h[(h - x)P_2] \quad (4.12) \]
where
\[ \theta_1 = (1 - \omega L)\phi_1 - \omega \epsilon L e_1 \quad \text{and} \quad \theta_2 = (1 - \omega L)\phi_2 - \omega \epsilon L e_2 \quad (4.13) \]
Substituting Eqs.(4.10) and (4.12) in Eq.(3.5), we obtain the stress function as follows:
\[ \bar{\sigma}(x, s) = \sigma_1 \sin h[(h - x)P_1] + \sigma_2 \sin h[(h - x)P_2] - \frac{D}{s} \quad (4.14) \]
where
\[ \sigma_1 = e_1 - \beta \theta_1 \quad \text{and} \quad \sigma_2 = e_2 - \beta \theta_2 \quad (4.15) \]
Now from Eq.(2.8) and Eq.(4.12) we can write the expression in the Laplace transform domain as,
\[ \frac{\partial \bar{u}}{\partial x} = e_1 \sin h[(h - x)P_1] + e_2 \sin h[(h - x)P_2] \quad (4.16) \]
Thus we have,
\[ \bar{u} = u_1 \cosh[(h - x)P_1] + u_2 \cosh[(h - x)P_2] \quad (4.17) \]
where
\[ u_1 = -\frac{e_1}{P_1} \quad \text{and} \quad u_2 = -\frac{e_2}{P_2} \quad (4.18) \]
which completes the solution.

5. Application of Boundary Conditions
Consider a finite rod \( 0 \leq x \leq h \) at a uniform temperature \( T_0 \).

5.1. Thermal Shock.
Here, we have applied the thermal shock to the boundary \( x = 0 \), given by
\[ F(0, t) = F_0 H(t) \quad (5.1) \]
where,
\[ H(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \]
is a Heaviside unit step function.
Taking Laplace transform of Eq.(5.1), we get,
\[ \phi_0 = \bar{F}(s) = \frac{F_0}{s} \quad (5.2) \]
After substituting value from Eq.(5.2) into Eqs.(4.8)–(4.18), we get the complete solution.
5.2. **Ramp-Type Heating.**

Here, we have applied the heating to the boundary \( x = 0 \), given by

\[
F(0, t) = \begin{cases} 
0, & t \leq 0 \\
\frac{F_0}{t_0} t, & 0 < t \leq t_0 \\
F_0, & t > t_0
\end{cases}
\]  

(5.3)

where, \( t_0 \) is the ramp-type parameter.

Taking Laplace transform of Eq.(5.3), we get,

\[
\phi_0 = \tilde{F}(s) = \frac{F_0 (1 - e^{-st_0})}{t_0 s^2}
\]  

(5.4)

After substituting value from Eq.(5.4) into Eqs.(4.8)–(4.18), we get the complete solution.

5.3. **Harmonically Varying Temperature.**

Here, we have applied the heating to the boundary \( x = 0 \), given by

\[
F(0, t) = F_0 e^{i \Omega t}
\]  

(5.5)

where, \( i = \sqrt{-1} \).

Taking the Laplace transform of Eq.(5.5), we obtain,

\[
\phi_0 = \tilde{F}(s) = \frac{F_0}{s - i \Omega}
\]  

(5.6)

After substituting value from Eq.(5.6) into Eqs.(4.8)–(4.18), we get the complete solution.

6. **Numerical Inversion of the Laplace Transform**

Let \( \hat{f}(r, z, s) \) be the Laplace transform of \( f(r, z, t) \). The inverse Laplace transform (Honig and Hirdes, 1984 [19]) is defined by,

\[
f(r, z, t) = \frac{e^{bt}}{2\pi} \int_{-\infty}^{\infty} e^{itv} \hat{f}(b + iv) dv
\]

where, \( b \) is a positive number such that \( b > \text{Re} \{ \text{singularities of } \hat{f}(r, z, s) \} \)

Here, the function \( f(r, z, t) \) is approximated by,

\[
f(r, z, t) \approx \frac{1}{2} a_0 + \sum_{k=1}^{N} a_k \approx f_N(r, z, t), \quad \text{for } 0 \leq t \leq 2l
\]

where,

\[
a_k = \frac{e^{bt}}{l} \text{Re} \left[ \frac{ik\pi t}{\sqrt{1 - e^{-l}}} \hat{f} \left( b + \frac{ik\pi}{l} \right) \right]
\]

7. **Numerical values of constants**

For the given problem we have taken the following values of numerical constants [35]:

\[
\begin{align*}
\epsilon &= 0.414774, & \beta &= 0.0025404, & \alpha &= 0.50, & \tau_0 &= 0.02, \\
D &= 10^{-7}, & F_0 &= 1.0, & \omega &= 0.1, & h &= 10, & \Omega &= 10^{-5}.
\end{align*}
\]
In this paper we have used the value $t = 0.25$ for calculating the numerical values of all field functions each for three different type of heating application and plotted the obtained results using MATLAB.

Figures 1–5 show the graphs of field functions with thermal shock for different values of $\omega = 0.0, 0.1$.

Figure 1 shows the fractional change in conductive temperature for fixed value of $\alpha = 0.50$. The function $\phi$ decreases as the value of $\omega$ increases from 0.0 to 0.1 as well as for the increase in value of $x$. It vanishes for large value of $x$.

Figure 2 shows the fractional change in thermodynamic temperature for fixed value of $\alpha = 0.50$. The function $\theta$ decreases for increase in value of $x$ and it vanishes for large value of $x$. Also it decreases when $\omega$ increases.

Figure 3 represents the displacement distribution for fixed fractional parameter $\alpha = 0.50$. The function $u$ decreases gradually when $x$ is increasing. Also as $\omega$ increases $u$ decreases.

Figure 4 represents the stress distribution for fixed value of $\alpha = 0.50$. The absolute value of $\sigma_{xx}$ decreases for $\omega = 0.0$ and $\omega = 0.1$ and vanishes for large value of $x$. Also as $\omega$ increases the absolute value of $\sigma_{xx}$ decreases.

Figure 5 shows the strain distribution for fixed value of $\alpha = 0.50$. Here, when $\omega = 0.0$ the absolute value of function $e$ takes maximum value $3.2 \times 10^{-3}$. It decreases as the value of $\omega$ increases. Also as $x$ increases value of $e$ gradually decreases and vanishes for large values of $x$. 

8. Graphical Interpretation
Figures 6–10 show the graphs of field functions with ramp-type heating for $t_0 = 0.1, 0.2, 0.3$.

Figure 6 represents the fractional change in $\phi$ for fixed value of $\alpha = 0.50$. Here, $\phi$ decreases as we go on increasing the value of $t_0$. The function $\phi$ decreases in each of three cases and vanishes for large value of $x$. 
Figure 7 represents the fractional change in $\theta$ for fixed value of $\alpha = 0.50$. Here $\theta$ decreases when $t_0$ is increasing. The function $\theta$ also decreases as $x$ is goes on increasing and vanishes for large $x$.

Figure 8 shows the fractional change in $u$ for fixed value of $\alpha = 0.50$. The function $u$ decreases for both cases when $t_0$ is increasing and $x$ is increasing.
Figure 9 represents the change in stress distribution for fixed value of fractional parameter \( \alpha = 0.50 \). Here there is a decrease in absolute value of \( \sigma_{xx} \) if \( t_0 \) goes on increasing as well as when \( x \) is increasing. Moreover, it vanishes for large \( x \).

Figure 10 represents the change in strain distribution for fixed value of fractional parameter \( \alpha = 0.50 \). The absolute value of \( e \) is maximum i.e., \( 3.1 \times 10^{-3} \) for \( t_0 = 0.1 \) and it decreases not only if \( t_0 \) increases but also for increase in \( x \). Moreover, it vanishes for large \( x \).

Figures 11–15 show the graphs of field functions with harmonically vary heating for different values of \( \tau_0 = 0.02, 0.12, 0.20 \)

Figure 11 shows the change in function \( \phi \) for fixed value of \( \alpha = 0.50 \). As we increase the value of \( \tau_0 \), the function \( \phi \) decreases. Also \( \phi \) goes on decreasing with increase in value of \( x \) and vanishes for large \( x \).

Figure 12 shows the change in function \( \theta \) for fixed value of \( \alpha = 0.50 \). Here, \( \theta \) decreases with increase in value of \( \tau_0 \) as well as it decreases with increase in value of \( x \) and vanishes for large \( x \).

Figure 13 represents the change in displacement function \( u \) for \( \alpha = 0.50 \). The function \( u \) increases when the value of \( \tau_0 \) increases, but \( u \) decreases gradually with increase in \( x \).

Figure 14 shows the change in stress function \( \sigma_{xx} \) for fixed value of \( \alpha = 0.50 \). The absolute value of \( \sigma_{xx} \) decreases when \( \tau_0 \) increases and also when \( x \) increases the function \( \sigma_{xx} \) decreases and vanishes for large \( x \).

Figure 15 shows the change in the strain distribution for fixed value of \( \alpha = 0.50 \). Here the absolute value of \( e \) is increases when \( \tau_0 \) increases but as \( x \) increases the value of \( e \) decreases and vanishes for large \( x \). It takes maximum absolute value \( 3.2 \times 10^{-3} \) for \( \tau_0 = 0.20 \).
9. Conclusion

We have solved the problem using two-type temperature method with three different heating applications and modified the work of E. Bassiouny and H. M. Youssef [35] applying the Caputo-Fabrizio fractional order derivative.

In this paper we can see that this theory has been recovered the discontinuities in the temperature, stress, strain and displacement. We observed that the fractional order parameter $\alpha$, 

\[ u \]
ramp-type parameter $t_0$ and the relaxation time $\tau_0$ has very significant effects on temperature, stress, strain and displacement.

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FRACTIONAL ORDER THERMOELASTIC PROBLEM FOR FINITE PIEZOELECTRIC ROD


