

FAST ANDROID IMPLEMENTATION OF MONTE CARLO SIMULATION FOR PRICING EQUITY-LINKED SECURITIES

HANBYEOL JANG¹, HYUNDONG KIM¹, SUBEOM JO², HANRIM KIM², SERI LEE³, JUWON LEE³,
AND JUNSEOK KIM^{1†}

¹DEPARTMENT OF MATHEMATICS, KOREA UNIVERSITY, SEOUL 02841, REPUBLIC OF KOREA
Email address: styliststar@korea.ac.kr, rlagusehd@korea.ac.kr, †cfdkim@korea.ac.kr

²DEPARTMENT OF FINANCIAL ENGINEERING, KOREA UNIVERSITY, SEOUL 02841, REPUBLIC OF KOREA

³DEPARTMENT OF CONTROL AND INSTRUMENTATION ENGINEERING, KOREA UNIVERSITY, SEJONG 30019, REPUBLIC OF KOREA

ABSTRACT. In this article, we implement a recently developed fast Monte Carlo simulation (MCS) for pricing equity-linked securities (ELS), which is most commonly issued autocallable structured financial derivative in South Korea, on the mobile platform. The fast MCS is based on Brownian bridge technique. Although mobile platform devices are easy to carry around, mobile platform devices are slow in computation compared to desktop computers. Therefore, it is essential to use a fast algorithm for pricing ELS on the mobile platform. The computational results demonstrate the practicability of Android application implementation for pricing ELS.

1. INTRODUCTION

Financial technology (FinTech) has become a popular emerging technical term which indicates new technologies used by the financial service companies [1]. One of the important works in pricing process of financial derivatives is to find the fair value. Also, an accurate, fast, and efficient numerical method is our interest when pricing of financial derivatives such as ELS product. Representatively, accurate, fast, and efficient numerical methods for ELS or option pricing are operator splitting method [2, 3], multigrid method [4], finite difference method [10], Monte Carlo method [5, 6, 7, 8, 9, 10], and Brownian bridge method [11]. Consequently, it is very important to develop mobile platform applications for fast financial option pricing. We consider a fast Android application for pricing three-asset equity-linked securities (ELS), which is one of most widely traded financial instruments in the Republic of Korea.

Received by the editors December 26 2019; Accepted March 16 2020; Published online March 25 2020.

Key words and phrases. Monte Carlo simulation, Brownian bridge technique, Equity-linked securities, option pricing.

[†] Corresponding author.

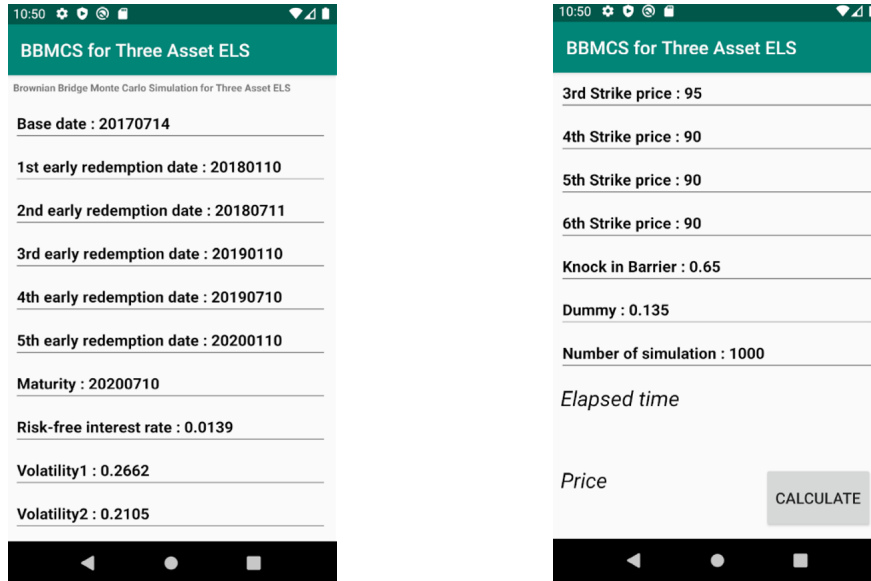


FIGURE 1. Emulator configuration for the fast ELS pricing algorithm.

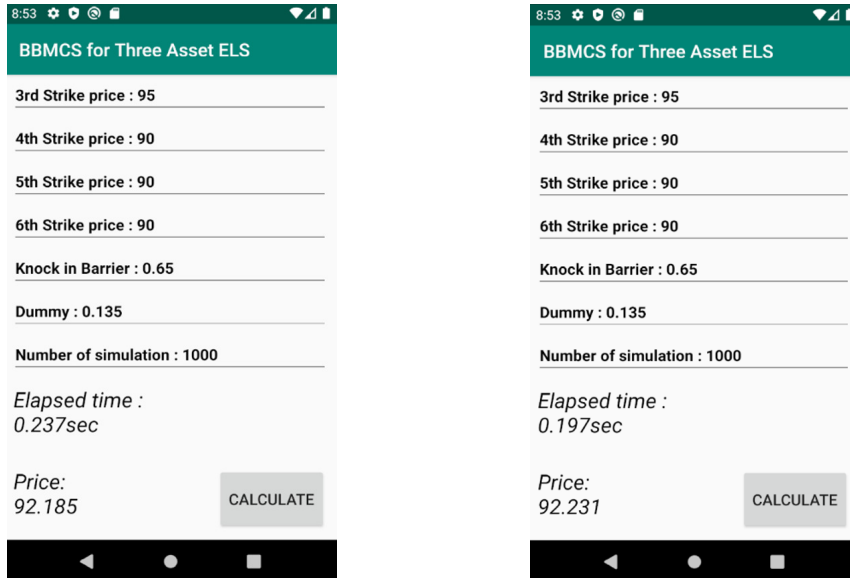


FIGURE 2. Pricing results in the emulator.

As an example of ELS payoff structure, let $K_1 > K_2 > K_3$ and $c_1 < c_2 < c_3$ be repayment criteria percentages and coupon rates at $T_1 < T_2 < T_3$, respectively. Let $X(t)$ be the scaled underlying asset value at time t . If $X(T_1) \geq K_1$, then the contract is ended with $(1 + c_1)F$,

TABLE 1. Comparison of the elapsed time (in seconds) for three-asset with strike prices $K_1 = 90$, $K_2 = 90$, $K_3 = 90$, $K_4 = 85$, $K_5 = 85$, $K_6 = 80$, knock-in barrier $D = 65$, volatilities $\sigma_1 = 0.25$, $\sigma_2 = 0.24$, $\sigma_3 = 0.23$, the correlation coefficient $\rho_{12} = 0.5$, $\rho_{13} = 0.5$, $\rho_{23} = 0.5$, the risk-free interest free $r = 0.02$.

M	10^3	5×10^3	10^4	5×10^4	10^5
BBMCS	0.1498	0.7474	1.5275	7.4854	14.8720
SMCS	1.7988	8.9995	17.8713	87.0325	177.2818
Ratio	12.0046	12.0403	11.6998	11.6269	11.9205

where F is the face value. Otherwise, if $X(T_2) \geq K_2$, then the contract is ended with $(1+c_2)F$. At T_3 , if $X(T_3) \geq K_3$, then the contract is closed with $(1+c_3)F$; otherwise, we check whether $\min_{0 \leq t \leq T_3} X(t) \leq \text{kib}$, where kib is a knock-in barrier, then $X(T_3)F/100$ is the return. Otherwise, the return is $(1+d)F$. For more details about the ELS payoff structure, we refer to [9]. The authors in [12, 14] constructed an application that calculates the price of an ELS in an Android-based smartphone and measured the elapsed time by the number of simulations using a Monte Carlo Simulation (MCS).

In this paper, we combine the two recent results to develop a fast Android application for pricing ELS: One is Android platform application [13] and the other is fast MCS for pricing three-asset equity-linked securities [9]. In [13], the authors developed an Android platform application for pricing ELS. However, computation is slow and it is difficult to be used in practice. In [9], the authors presented a fast MCS using MATLAB platform not Android application. For all computational simulation, ELS data set in derivatives of miraeassetdaewoo is used.

This paper is organized as follows. In Section 2, we briefly review the fast MCS algorithm. In Section 3, we present the results of numerical experiments for a three-asset ELS.

2. FAST MONTE CARLO SIMULATION

Algorithm 1 shows the fast MCS algorithm which is based on the Brownian bridge technique for the three-asset ELS as shown in the reference [9]. This algorithm is short and concise by using the Monte Carlo method and Brownian bridge technique. Therefore, compared to the Standard MCS (SMCS) algorithm, the computational performance is about 12 times faster. The comparison results between elapsed times of SMCS and BBMCS show in Table 1.

3. NUMERICAL RESULTS AND CONCLUSION

Now, we perform numerical tests such as the convergence test and the comparison test of the computation time between the SMCS and the fast MCS. All computations are run in emulator API level 28 in Android Studio with 8GB RAM. Figure 1 shows emulator configuration for Brownian bridge Monte Carlo simulation (BBMCS) of ELS pricing algorithm. Figure 2 shows pricing results in the emulator.

Algorithm 1 Fast MCS algorithm for three-asset ELS

Require: Set initial price $X(0) = X_0$, maturity T , the number of checking days N_c , the number of sample paths N_m , the number of total time steps N_T , time-step size $\Delta t = \frac{T}{N_T}$, face value F , volatilities σ_n of underlying assets, risk-neutral interest rate r , early redemption dates T_i , coupon rates c_i for early and final redemptions, strike percentages K_i , dummy d , and knock-in barrier kib. Set $M_i = 0$ and $X_n(0) = 100$ and worst performer $WP(t) = \min(X_1(t), X_2(t), X_3(t))$. Here, $1 \leq i \leq N_c$ and $T_0 = 0$.

for $k = 1$ to N_m **do**

▷ Generate stock path for T_i only as

for $i = 0$ to $N_c - 1$ **do**

$$X_n(T_{i+1}) = X_n(T_i) \exp((r - 0.5\sigma^2)(\Delta T_{i+1}) + \sigma_n \sqrt{\Delta T_{i+1}} Z_n^*), \quad Z_n^* \sim N(0, 1)$$

end for

▷ Check the value of the stock path at checking days

if $WP(T_1) \geq K_1$ **then** $M_1 = M_1 + (1 + c_1)F$

else if $WP(T_2) \geq K_2$ **then** $M_2 = M_2 + (1 + c_2)F$

⋮

else if $WP(T_{N_c}) \geq K_{N_c}$ **then** $M_{N_c} = M_{N_c} + (1 + c_{N_c})F$

else if $\min_{1 \leq i \leq N_c} WP(T_i) \leq \text{kib}$ **then** $M_{N_c} = M_{N_c} + \frac{X(T_{N_c+1})}{100}F$

else

▷ Generate a daily stock path passing through X as

for $i = 0$ to $N_c - 1$ **do**

Set $Y_n(T_i) = X_n(T_i)$

for $j = \frac{T_i}{\Delta t}$ to $\frac{T_{i+1}}{\Delta t} - 1$ **do**

$$Y_n(t_{j+1}) = Y_n(t_j) \exp(w_n(t_j)),$$

where $w_n(t_j) = (r - 0.5\sigma^2)\Delta t + \sigma_n \sqrt{\Delta t} Z_n^*$, $Z_n^* \sim N(0, 1)$, and $t_j = j\Delta t$

end for

▷ Apply the BB

for $j = \frac{T_i}{\Delta t}$ to $\frac{T_{i+1}}{\Delta t} - 1$ **do**

$$Y_n(t_{j+1}) = Y_n(T_i) \exp(W_n(t_j)), \quad W_n(t_j) = \sum_{p=\frac{T_i}{\Delta t}}^j w(t_p)$$

end for

for $j = \frac{T_i}{\Delta t}$ to $\frac{T_{i+1}}{\Delta t} - 1$ **do**

$$X_n(t_j) = X_n(T_i) \exp(B_n(t_j)), \quad B_n(t_j) = W_n(t_j) + \frac{t_j - T_i}{\Delta T_{i+1}} \log \frac{X_n(T_{i+1})}{Y_n(T_{i+1})}$$

$$WP(t_j) = \min(X_1(t_j), X_2(t_j), X_3(t_j))$$

end for

end for

if $\min_{1 \leq j \leq \frac{N_c}{\Delta t}} \{WP(t_j)\} \leq \text{kib}$ **then** $M_{N_c} = M_{N_c} + F X_n(T_{N_c+1})/100$

else

$$M_{N_c} = M_{N_c} + (1 + d)F$$

end if

end if

end for

▷ Take average and discount to present value. $V^0 = \sum_{i=1}^{N_c} e^{-rT_i} M_i / N_m$

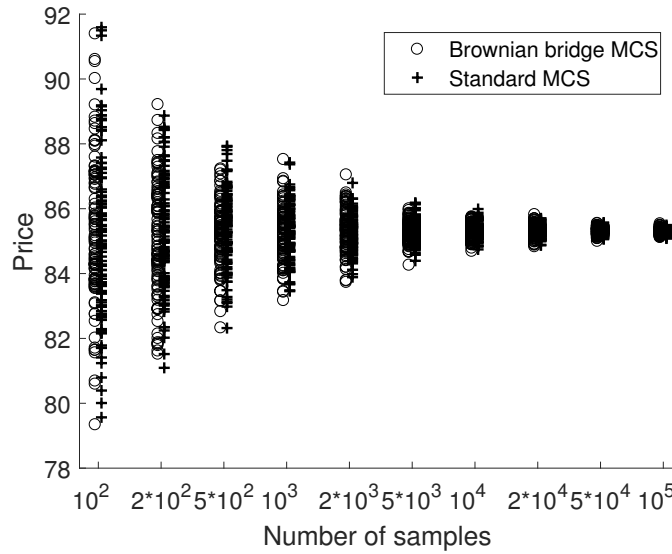


FIGURE 3. Comparison of the elapsed time (in seconds) between SMCS and fast MCS in three-asset ELS pricing.

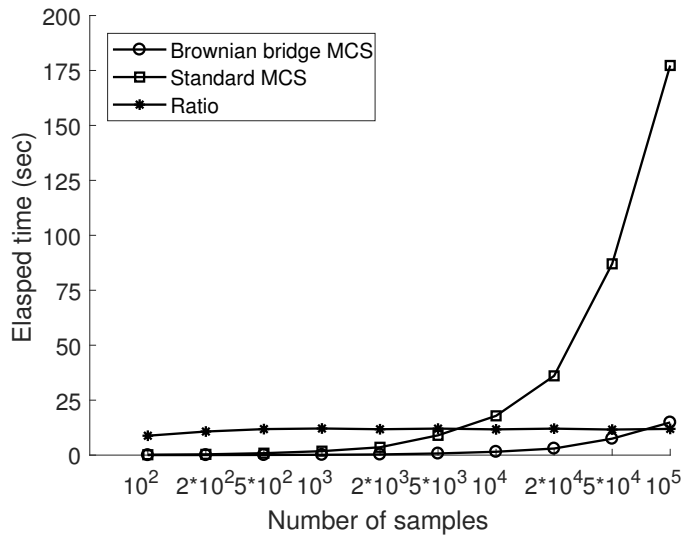


FIGURE 4. ELS price versus the number of samples. Here, we plot 100 simulation results for each case in three-asset.

Table 1 shows comparison results of elapsed times. This demonstrates that the fast MCS is approximately 12 times faster than SMCS.

Figure 3 shows comparison of the elapsed time (in seconds) between SMCS and fast MCS in three-asset ELS pricing.

Finally, Fig. 4 shows ELS price versus the number of samples and we can confirm that the results converge.

ACKNOWLEDGMENT

The corresponding author (J.S. Kim) was supported by the Brain Korea 21 Plus (BK 21) from the Ministry of Education of Korea. The authors would like to thank the reviewers for their constructive and helpful comments on the revision of this article.

REFERENCES

- [1] K Gai, M. Qiu, and X. Sun, *A survey on FinTech*, J. Netw. Comput. Appl. **103** (2018), 262–273.
- [2] D. Jeong, I. S. Wee, and J. Kim, *An operator splitting method for pricing the ELS option*, J. KSIAM. **14** (2010), 175–187.
- [3] D. Jeong, and J. Kim, *A comparison study of ADI and operator splitting methods on option pricing models*, J. Comput. Appl. Math. **247** (2013), 162–171.
- [4] D. Jeong, Y. Li, Y. Choi, K. S. Moon, and J. Kim, *An adaptive multigrid technique for option pricing under the Black-Scholes model*, J. KSIAM. **17** (2013), 295–306.
- [5] P. P. Boyle, *Options: A Monte Carlo approach*, J. financ. econ. **4** (1977), 323–338.
- [6] P. Glasserman, *Monte Carlo methods in financial engineering*, Springer Science & Business Media, **53** (2013).
- [7] Á. Leitao, L. A. Grzelak, and C. W. Oosterlee, *On a one time-step Monte Carlo simulation approach of the SABR model: Application to European options*, Appl. Math. Comput. **293** (2017), 461–479.
- [8] G. Bormetti, G. Callegaro, G. Livieri, A. Pallavicini, *A backward Monte Carlo approach to exotic option pricing*, Eur. J. Appl. Math. **29** (2018), 146–187.
- [9] H. Jang, S. Kim, J. Han, S. Lee, J. Ban, H. Han, C. Lee, D. Jeong, and J. Kim, *Fast Monte Carlo simulation for pricing Equity-Linked Securities*, Comput. Econ. (2019). <https://doi.org/10.1007/s10614-019-09947-2>
- [10] D. Jeong, M. Yoo, C. Yoo, J. Kim, *A hybrid monte carlo and finite difference method for option pricing*, Comput. Econ. **53** (2019), 111–124.
- [11] S. A. Metwally, A. F. Atiya, *Using Brownian bridge for fast simulation of jump-diffusion processes and barrier options*, J. Deriv. **10** (2002), 43–54.
- [12] W. Jian, J. Ban, J. Han, S. Lee, and D. Jeong, *Mobile platform for pricing of Equity-Linked Securities*, J. KSIAM. **21** (2017), 181–202.
- [13] H. Jang, H. Han, H. Park, W. Lee, J. Lyu, J. Park, H. Kim, C. Lee, S. Kim, Y. Choi, and J. Kim, *Android application for pricing two-and three-asset Equity-Linked Securities*, J. KSIAM. **23** (2019), 237–251.
- [14] H. L. Koh, and S. Y. Teh, *Learning Black Scholes option pricing the fun way via mobile apps*, In Proceedings of 2013 IEEE International Conference on Teaching, Assessment and Learning for Engineering (TALE), (2013), 192–195.