A Piecewise Constant Staggered DG Method for the Biharmonic Problem

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ABSTRACT

In this poster presentation, we have developed a staggered discontinuous Galerkin(SDG) method for the biharmonic equation $\Delta^2 u = f$. To apply SDG, we should rewrite it as a system of first order equations with the auxiliary variables $\boldsymbol{p} = -\nabla u$, $z = \nabla \cdot \boldsymbol{p}$ and $\boldsymbol{\sigma} = -\nabla z$, and then divide the primal meshes into the dual meshes. This is the attractive method in that the application of the method is flexible to the distorted grids. We have proven the a priori error analysis for all the variables, and the superconvergence of the approximation of the primal variable to an appropriate projection of the primal variable without additional regularity more than $z \in H^1(\Omega)$. Next, we introduce an adaptive mesh refinement, highly suitable to treat hanging nodes. We derive three residual-type error estimators measured in L^2 errors of $\boldsymbol{\sigma}$, z and $\boldsymbol{p}$, respectively. Finally, there are some numerical experiments to confirm the theories.

SDG METHOD

Dual meshes for the primal meshes

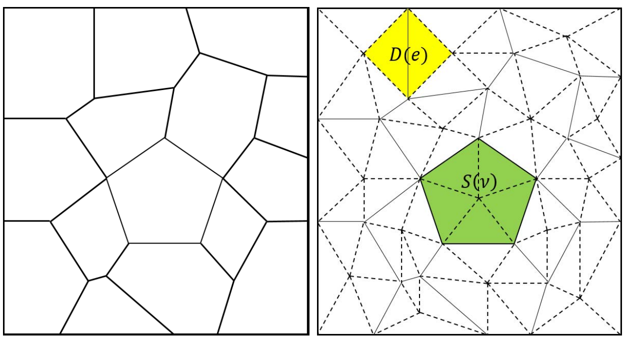

Figure 1. The primal meshes(left) and the corresponding dual meshes(right)

Function Spaces and Norms

The discrete space for the scalar variable

$$S_h := \{v\,|_\tau \in P^0(\tau),\ [v]\,|_e = 0\ \forall e \in \mathcal{F}_{prm}\}. \tag{1}$$

The discrete space for the vector variable

$$V_h := \{\boldsymbol{q}\mid_\tau \in P^0(\tau)^2,\ [\boldsymbol{q}\cdot\boldsymbol{n}]\mid_e = 0\ \forall e \in \mathcal{F}_{dl}\}. \tag{2}$$

A PRIORI ERROR ANALYSIS

Error Estimate

We have the following first order convergences.

$$\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_h\|_0 + \|z-z_h\|_0 + \|\boldsymbol{p}-\boldsymbol{p}_h\|_0 + \|u-u_h\|_0 \le Ch, \tag{3}$$

Superconvergence

If $z \in H^1(\Omega)$, the approximation of u superconverges to the special projection of u.

$$\|\mathcal{I}_h u - u_h\|_0 \le Ch^2. \tag{4}$$

ADAPTIVE MESH REFINEMENT

A posteriori error estimators

$$\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_h\|_0 \le C\eta_s, \tag{5}$$
$$\|z-z_h\|_0 \le C\eta_z, \tag{6}$$
$$\|\boldsymbol{p}-\boldsymbol{p}_h\|_0 \le C\eta_p, \tag{7}$$

where η's are the proposed residual type estimators.

REFERENCES

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