Subcell Monotonic Limiting Strategy for Higher-order Methods on Two-dimensional Mixed Meshes

Hojun You [1], Chongam Kim[1,2]

1) *Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul 08826, KOREA*
2) *Institute of Advanced Aerospace Technology, Seoul National University, Seoul 08826, KOREA*

Corresponding Author: Hojun You, rememory@snu.ac.kr

ABSTRACT

The present paper deals with a new improvement of hierarchical multi-dimensional limiting process (hMLP) for resolving the subcell distribution of high-order methods on two dimensional mixed meshes. It was reported that hMLP has several remarkable characteristics such as the capability to preserve the formal order-of-accuracy and to capture discontinuities efficiently and accurately. However, such characteristics are valid only on simplex meshes, and numerical Gibbs-Wilbraham oscillations are concealed in high-order polynomial modes. To address these two issues, we introduce 1) the concept of simplex-decomposition and 2) the novel troubled-boundary detector into the framework of hMLP, leading to the design of hMLP_BD. The behavior of hMLP on mixed meshes is rigorously examined, and the simplex-decomposed MLP condition and smooth extrema detector are derived for mixed meshes. The troubled-boundary detector is designed by analyzing the behavior of computed solutions across boundary-edges to capture subcell Gibbs-Wilbraham oscillations. Through extensive numerical tests, it is confirmed that hMLP_BD successfully eliminates spurious subcell oscillations and provides reliable subcell distributions on two-dimensional mixed grids, while preserving the expected order-of-accuracy in smooth regions.

NUMERICAL RESULTS

We show the computed results of the strong vortex and shock wave interaction problem. This test aims to assess the subcell resolution of complex flow patterns arising from the interaction between a strong vortex and a shock wave. This is also one of the benchmark problems of the 5th International Workshop on High Order CFD Methods (HiOCFD5). The computational domain is $[0,2] \times [0,1]$, and a normal stationary shock wave with $M_s = 1.5$ is initially located at $x = 0.5$. The flow is from the left to the right direction with the upstream condition given by $(\rho_0, u_0, v_0, p_0) = (1, M_s\sqrt{\gamma}, 0, 1)$, where the specific heat ratio, γ, is 1.4. A strong isentropic vortex with the following angular velocity is considered:

$$w_\theta = \begin{cases} w\dfrac{r}{a}, & r \le a, \\ w\dfrac{a}{a^2 - b^2}\left(r - \dfrac{b^2}{r}\right), & a < r \le b, \\ 0, & r > b, \end{cases} \tag{1}$$

where r is the distance from the center of the vortex core located at $(0.25, 0.5)$. Here, the radii a and b are $(a, b) = (0.075, 0.175)$, and the vortex strength is $M_{\mathrm{v}} = w/a_{\mathrm{U}} = 0.9$, where a_{U} is the speed of sound on the upstream side. We used both irregular triangular (IT) and mixed (IM) meshes, where the characteristic size of elements is $h = 1/300$. The discontinuous Galerkin method with $P3$ approximation is used with the 4$^{\mathrm{th}}$ order 5 stages strong stability preserving Runge-Kutta time integration under the condition of $\mathrm{CFL} = 0.9$. Figure 1 shows the subcell distributions of density contours at $t = 0.7$. The splitting of the incoming vortex after passing the stationary shock is accurately captured. On top of it, hMLP_BD effectively controls the subcell overshoots and undershoots across the stationary shock. As a result, small-scale numerical artifacts on the downstream flow fields, triggered by large subcell Gibbs-Wilbraham oscillations across the shock, are significantly eliminated. During the simulation, an additional pressure scaling technique is required for hMLP to prevent negative pressure values, while it is unnecessary for hMLP_BD, resulting in 22.4% and 20.5% reductions in the computational cost by hMLP_BD on the IT and IM meshes, respectively.

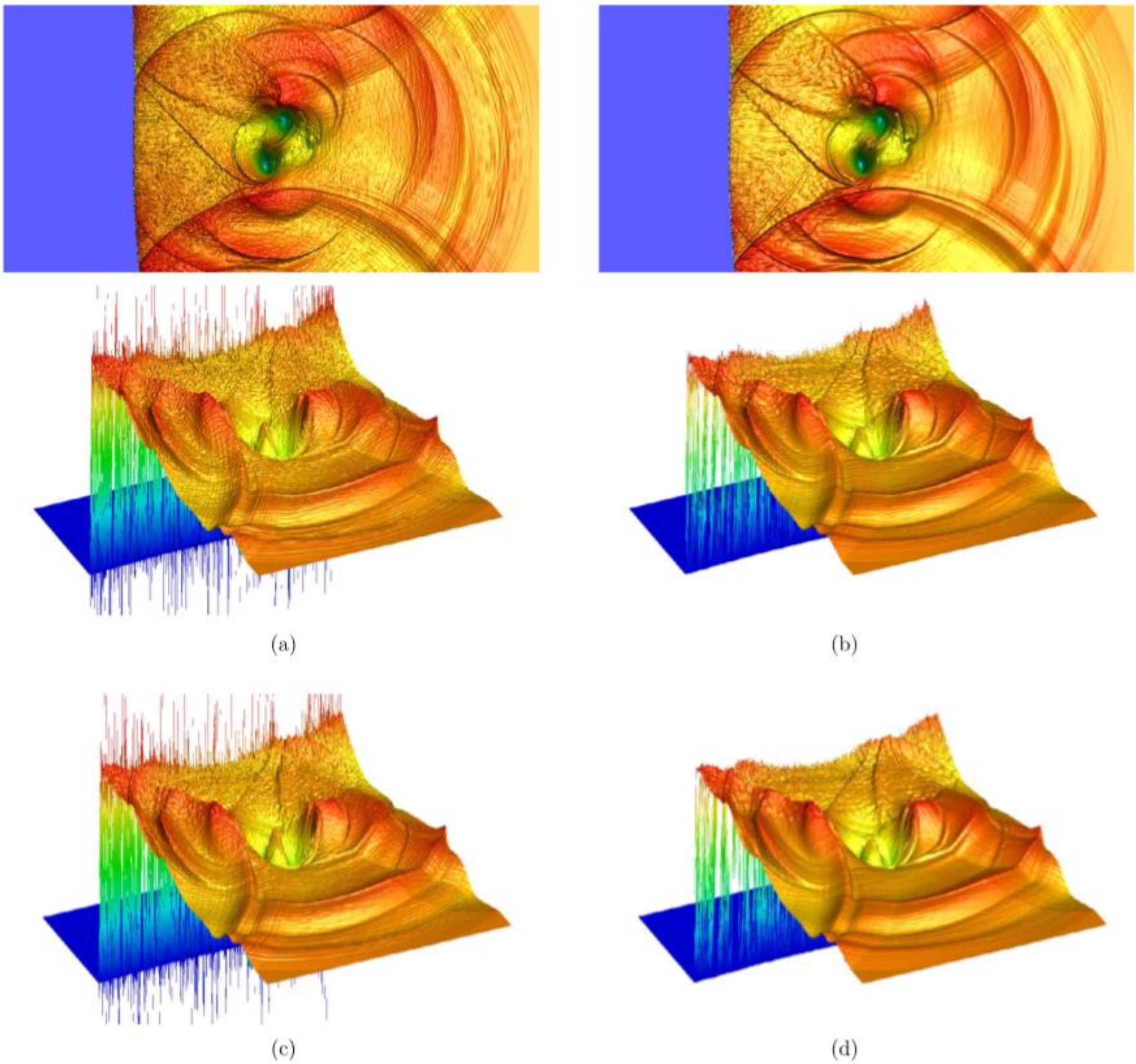

Figure 1. 3-D perspectives of density contours of the strong vortex and shock wave interaction problem to highlight subcell Gibbs-Wilbraham oscillations ($P3$ approximation): (a) IT mesh with hMLP, (b) IT mesh with hMLP_BD, (c) IM mesh with hMLP, and (d) IM mesh with hMLP_BD.

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REFERENCES

1. You, H. and Kim, C., "High-order multi-dimensional limiting strategy with subcell resolution I. Two-dimensional mixed meshes," *Journal of Computational Physics*, Vol. 375, 2018, pp. 1005-1032.
2. Park, J.S. and Kim, C., "High-order multi-dimensional limiting strategy for discontinuous Galerkin methods in compressible inviscid and viscous flows," *Computers & Fluids*, Vol. 96, 2014, pp. 377-396.
3. Park, J.S., You, H. and Kim, C., "High-order multi-dimensional limiting process for DG and FR/CPR methods on tetrahedral meshes," *Computers & Fluids (Invited special issue from ICCFD8)*, Vol. 154, 2017, pp. 322-334.
4. You, H. and Kim, C., "Higher-order multi-dimensional limiting strategy for subcell resolution," Proc. of the 23rd American Institute of Aeronautics and Astronautics Computational Fluid Dynamics Conference, AIAA2017-4110, Denver U.S., June 2017.