INFLUENCE OF HALL CURRENT AND HEAT SOURCE ON MHD FLOW OF A ROTATING FLUID IN A PARALLEL POROUS PLATE CHANNEL

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ABSTRACT. This paper examined the MHD and thermal behavior of unsteady mixed convection flow of a rotating fluid in a porous parallel plate channel in the presence of Hall current and heat source. The exact solutions of the concentration, energy and momentum equations are obtained. The influence of each governing parameter on non dimensional velocity, temperature, concentration, skin friction coefficient, rate of heat transfer and rate of mass transfer at the porous parallel plate channel surfaces is discussed. During the course of numerical computation, it is observed that as Hall current parameter and Soret number at the porous channel surfaces increases, the primary and secondary velocity profiles are increases while the primary and secondary skin friction coefficients are increases at the cold wall and decreases at the heated wall. In particular, it is noticed that a reverse trend in case of heat source parameter.

1. INTRODUCTION

Investigation of MHD flows in a rotating system is of great importance because of its wide range industrial applications in rotating MHD generators, turbo machines, rotating drum-type separators for liquid metal MHD applications, electromagnetic stirring of liquid metal in continuous casting machines. It is observed that the Coriolis force is stronger than inertia and viscous forces whereas Coriolis and magnetic forces are comparable in magnitude. In addition, the Coriolis force induces a secondary flow in the flow field. Given the importance of this study, unsteady MHD mixed convection flow in a rotating system has been investigated by several researchers considering different aspects of the problem. Sharma and Chaudhary [1]...

The convection problem in porous media has important applications in geothermal reservoirs, geothermal energy extractions, coal gasification, iron blast furnaces, ground water hydrology, wall cooled catalytic reactors, energy efficient drying processes, solar power collectors, cooling of nuclear fuel in shipping flasks, cooling of electronic equipments and natural convection in earth's crust. Venkateswarlu et al. [13] studied the diffusion-thermo effects on MHD flow past an infinite vertical porous plate in the presence of radiation and chemical reaction. Bhatti et al. [14] presented the mathematical modeling of heat and mass transfer effects on MHD peristaltic propulsion of two-phase flow through a Darcy-Brinkman-Forchheimer porous medium. Ellahi et al. [15] discussed the combine porous and magnetic effect on some fundamental motions of Newtonian fluids over an infinite plate. Venkateswarlu and Makinde [16] presented the unsteady MHD slip flow with radiative heat and mass transfer over an inclined plate embedded in a porous medium. Das et al. [17] considered the Hall effects on unsteady rotating MHD flow through porous channel with variable pressure gradient. Recently, Venkateswarlu et al. [18, 19, 20] reported the Soret and Dufour effects on radiative MHD slip flow of a viscous fluid in a parallel porous plate channel under the influence of heat absorption and chemical reaction.

The following strategy is pursued in the rest of the paper. Section two presents the formation of the problem. The analytical solutions are presented in section three. Results are discussed in section four and finally section five provides a conclusion of the paper.
Consider the non linear, unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting, heat absorbing and rotating fluid between two infinite parallel porous walls \( y = 0 \) and \( y = a \) in the presence of a uniform transverse magnetic field of strength \( B_0 \) that is applied parallel to \( y- \) axis taking Hall current into account. Select the coordinate system in such way that \( x- \) axis is along the length of the plate in the horizontal direction, \( y- \) axis is along the channel width in the vertical direction and \( z- \) axis perpendicular to \( xy- \) plane. Both the fluid and channel rotate in unison with a uniform angular velocity \( \Omega \) about \( y- \) axis. Fluid flow within the channel is induced due to uniform pressure gradient applied along \( x- \) direction as well as the movement of upper wall \( y = a \) with uniform velocity \( u_0 \) in the same direction. Initially i.e. at \( y = 0 \), both the fluid and plate are at rest and maintained at uniform temperature \( T = T_0 \) and uniform concentration \( C = C_0 \). At \( y = a \), temperature of the plate is raised to uniform temperature \( T = T_1 \). Also, species concentration at the surface of the plate is raised to uniform species concentration \( C = C_1 \). Physical model of the problem is presented in Fig. 1. Since channel walls are of infinite extent in \( x- \) and \( z- \) directions, all physical quantities, except pressure gradient depend on \( y \) and \( t \) only. It is assumed that the induced magnetic field produced by fluid motion is negligible in comparison to the applied one.

Keeping in view of the above assumptions, the governing equations for non linear, unsteady, hydromagnetic mixed convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid in a in a rotating system taking Hall current and Soret effects into account are presented in the following form.
Continuity equation:
\[ \frac{\partial v}{\partial y} = 0 \]  

(2.1)

Momentum equations:
\[ \frac{\partial u}{\partial t} + 2\Omega w = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left[ \frac{u + mw}{1 + m^2} \right] + g\beta_T (T - T_0) + g\beta_C (C - C_0) - \frac{\nu}{K_1} u \]  

(2.2)

\[ \frac{\partial w}{\partial t} - 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left[ \frac{mu - w}{1 + m^2} \right] - \frac{\nu}{K_1} w \]  

(2.3)

Energy equation:
\[ \frac{\partial T}{\partial t} = k_T \frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho c_p} (T - T_0) \]  

(2.4)

Concentration equation:
\[ \frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \]  

(2.5)

where \( m = \omega_e \tau_e \) – Hall current parameter, \( \omega_e \) – cyclotron frequency, \( \tau_e \) – electron collision time, \( u \) – fluid velocity in \( x \) – direction, \( v \) – fluid velocity in \( y \) – direction, \( w \) – fluid velocity in \( z \) – direction, \( p \) – fluid pressure, \( g \) – acceleration due to gravity, \( \rho \) – fluid density, \( \beta_T \) – coefficient of thermal expansion, \( \beta_C \) – coefficient of concentration expansion, \( t \) – time, \( K_1 \) – permeability of porous medium, \( B_0 \) – magnetic induction, \( T \) – fluid temperature, \( T_0 \) – temperature at the cold wall, \( k_T \) – thermal diffusivity of the fluid, \( Q_0 \) – dimensional heat source parameter, \( C \) – species concentration in the fluid, \( C_0 \) – concentration at the cold wall, \( \sigma \) – fluid electrical conductivity, \( c_p \) – specific heat at constant pressure, \( D_m \) – chemical molecular diffusivity, \( T_m \) – mean fluid temperature and \( \nu \) – kinematic viscosity of the fluid respectively.

Assuming that no slipping occurs between the plate and fluid, the corresponding initial and boundary conditions of the system of partial differential equations are given below

\[ u = 0, \ w = 0, \ T = T_0, \ C = C_0 \ at \ y = 0 \]
\[ u = u_0, \ w = 0, \ T = T_1 + \epsilon (T_1 - T_0) \ exp(int), \]
\[ C = C_1 + \epsilon (C_1 - C_0) \ exp(int) \ at \ y = a \]  

(2.6)

where \( T_1 \) – fluid temperature at the heated plate, \( C_1 \) – species concentration at the heated plate, \( n \) – frequency of oscillation and \( \epsilon << 1 \) is a very small positive constant.

For purely an oscillatory flow, we take the pressure gradient terms \(-\frac{1}{\rho} \frac{\partial p}{\partial x}\) and \(-\frac{1}{\rho} \frac{\partial p}{\partial z}\) are of the form (see, Makinde et al. [21])

\[ \frac{\partial p}{\partial x} = -\frac{\sigma B_0^2 u_0}{(1 + m^2)} \]  

(2.7)

\[ \frac{\partial p}{\partial z} = 2\Omega u_0 \rho + \frac{\sigma B_0^2 m u_0}{(1 + m^2)} \]  

(2.8)

By substituting the equations (2.7) and (2.8) into the equations (2.2) and (2.3) we get
\[
\frac{\partial u}{\partial t} + 2\Omega w = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left[ \frac{u + mw - u_0}{1 + m^2} \right] + g\beta_T (T - T_0) + g\beta_C (C - C_0) - \frac{\nu}{K_1} u \tag{2.9}
\]

\[
\frac{\partial w}{\partial t} - 2\Omega(u - u_0) = \nu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left[ \frac{mu_0 + w - mu}{1 + m^2} \right] - \frac{\nu}{K_1} w \tag{2.10}
\]

We introduce the following non-dimensional variables

\[
\eta = \frac{y}{a}, U = \frac{u}{\nu}, W = \frac{w}{\nu}, \omega = \frac{a^2}{\nu} \eta, \tau = \frac{t}{\nu}, \theta = \frac{T - T_0}{T_1 - T_0}, \phi = \frac{C - C_0}{C_1 - C_0} \tag{2.11}
\]

Equations (2.4), (2.5), (2.9) and (2.10) reduces to the following non-dimensional form

\[
\frac{\partial U}{\partial \tau} + 2K_2^2 W = \frac{\partial^2 U}{\partial \eta^2} - \frac{M(U + mW - \lambda)}{1 + m^2} + Gr\theta + Gm\phi - \frac{U}{K} \tag{2.12}
\]

\[
\frac{\partial W}{\partial \tau} - 2K_2^2(U - \lambda) = \frac{\partial^2 W}{\partial \eta^2} - \frac{M(m\lambda + W - mU)}{1 + m^2} - \frac{W}{K} \tag{2.13}
\]

\[
\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} - H\theta \tag{2.14}
\]

\[
\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial \eta^2} + S\frac{\partial^2 \theta}{\partial \eta^2} \tag{2.15}
\]

Here \( K^2 = \frac{\Omega a^2}{\nu} \) is the rotation parameter, \( M = \frac{\sigma B_0^2 a^2}{\rho \nu^2} \) is the magnetic parameter, \( \lambda = \frac{u_0a}{\nu} \) is the upper wall motion parameter, \( K = \frac{K_1 a}{\nu} \) is the permeability parameter, \( Gr = \frac{g\beta_T (T_1 - T_0) a^3}{\nu^2} \) is the thermal buoyancy force, \( Gm = \frac{g\beta_C (C_1 - C_0) a^3}{\nu^2} \) is the concentration buoyancy force, \( Pr = \frac{\nu p c}{\kappa} \) is the Prandtl number, \( H = \frac{Q a^2}{\rho C^2} \) is the heat source parameter, \( Sc = \frac{\nu}{D_m} \) is the Schmidt number and \( Sr = \frac{D_m k (T_1 - T_0)}{T_m (C_1 - C_0)} \) is the Soret number respectively.

The corresponding initial and boundary conditions can be written as

\[
U = 0, W = 0, \theta = 0, \phi = 0 \text{ at } \eta = 0
\]

\[
U = \lambda, W = 0, \theta = 1 + \epsilon \exp(i\omega\tau), \phi = 1 + \epsilon \exp(i\omega\tau) \text{ at } \eta = 1 \tag{2.16}
\]

Equations (2.12) and (2.13) can be written in compact form as follows

\[
\frac{\partial \psi}{\partial \tau} - 2iK_2^2(\psi - \lambda) = \frac{\partial^2 \psi}{\partial \eta^2} - \frac{M(\psi - \lambda)}{1 + im} + Gr\theta + Gm\phi - \frac{\psi}{K} \tag{2.17}
\]

The corresponding initial and boundary conditions in compact form can be written as

\[
\psi = 0, \theta = 0, \phi = 0 \text{ at } \eta = 0
\]

\[
\psi = \lambda, \theta = 1 + \epsilon \exp(i\omega\tau), \phi = 1 + \epsilon \exp(i\omega\tau) \text{ at } \eta = 1 \tag{2.18}
\]

Given the velocity, temperature and concentration fields in the boundary layer, the shear stress \( \tau_w \), the heat flux \( q_w \) and mass flux \( j_w \) are obtained as
\[ \tau_w = \mu \left[ \frac{\partial \psi}{\partial y} \right] \]  
\[ q_w = -k_T \left[ \frac{\partial T}{\partial y} \right] \]  
\[ j_w = -D_m \left[ \frac{\partial C}{\partial y} \right] \]

In non-dimensional form the skin-friction coefficient \( Cf \), heat transfer coefficient \( Nu \) and mass transfer coefficient \( Sh \) are defined as

\[ Cf = \frac{\tau_w}{\rho \left( \frac{U}{a} \right)^2} \]  
\[ Nu = \frac{aq_w}{k_T(T_1 - T_0)} \]  
\[ Sh = \frac{aj_w}{D_m(C_1 - C_0)} \]

Using non-dimensional variables in equation (2.11) and equations (2.19) to (2.21) into equations (2.22) to (2.24), we obtain the physical parameters

\[ Cf = \left[ \frac{\partial \psi}{\partial \eta} \right] \]  
\[ Nu = -\left[ \frac{\partial \theta}{\partial \eta} \right] \]  
\[ Sh = -\left[ \frac{\partial \phi}{\partial \eta} \right] \]

3. Solution of the Problem

Equations (2.14), (2.15) and (2.17) are coupled non-linear partial differential equations and these cannot be solved in closed form. So, we reduce these non-linear partial differential equations into a set of ordinary differential equations, which can be solved analytically. This can be done by assuming the trial solutions for the velocity, temperature and concentration of the fluid as (see, Adesanya and Makinde [22], Venkateswarlu and Venkata Lakshmi [23])

\[ \psi(\eta, \tau) = \psi_0(\eta) + \epsilon \exp(i\omega \tau)\psi_1(\eta) + o(\epsilon^2) \]  
\[ \theta(\eta, \tau) = \theta_0(\eta) + \epsilon \exp(i\omega \tau)\theta_1(\eta) + o(\epsilon^2) \]  
\[ \phi(\eta, \tau) = \phi_0(\eta) + \epsilon \exp(i\omega \tau)\phi_1(\eta) + o(\epsilon^2) \]

Substituting equations (3.1) to (3.3) into equations (2.14), (2.15) and (2.17), then equating the harmonic and non-harmonic terms and neglecting the higher order terms of \( o(\epsilon^2) \), we obtain...
\[ \psi_0'' - \left[ \frac{M}{1 + im} + \frac{1}{K} - 2iK^2 \right] \psi_0 = -Gr\theta_0 - Gm\phi_0 - \left[ \frac{M}{1 + im} - 2iK^2 \right] \lambda \] (3.4)

\[ \psi_1'' - \left[ \frac{M}{1 + im} + \frac{1}{K} - 2iK^2 + i\omega \right] \psi_1 = -Gr\theta_1 - Gm\phi_1 \] (3.5)

\[ \theta_0'' - PrH\theta_0 = 0 \] (3.6)

\[ \theta_1'' - Pr[H + i\omega]\theta_1 = 0 \] (3.7)

\[ \phi_0'' = -ScSr\theta_0' \] (3.8)

\[ \phi_1'' - Sc i \omega \phi_1 = -ScSr\theta_1' \] (3.9)

where the prime denotes the ordinary differentiation with respect to \( \eta \).

The corresponding initial and boundary conditions can be written as

\[ \psi_0 = 0, \quad \psi_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0, \quad \phi_0 = 0, \quad \phi_1 = 0, \quad \text{at} \quad \eta = 0 \]

\[ \psi_0 = \lambda, \quad \psi_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1, \quad \text{at} \quad \eta = 1 \] (3.10)

The analytical solutions of equations (3.4) to (3.9) with the boundary conditions in equation (3.10), are given by

\[ \psi_0 = \lambda a_{13}[1 - \exp(-a_{10}\eta)] + a_{12}\theta_0 + \frac{a_{16}\sinh a_{10}\eta}{\sinh a_{1}} + \frac{a_{11}\sinh a_{1}\eta}{\sinh a_{1}} \] (3.11)

\[ \psi_1 = \frac{a_{20}\sinh a_{17}\eta}{\sinh a_{17}} + \frac{a_{18}\sinh a_{2}\eta}{\sinh a_{2}} - \frac{a_{19}\sinh a_{5}\eta}{\sinh a_{5}} \] (3.12)

\[ \theta_0 = \frac{\sinh a_{1}\eta}{\sinh a_{1}} \] (3.13)

\[ \theta_1 = \frac{\sinh a_{2}\eta}{\sinh a_{2}} \] (3.14)

\[ \phi_0 = a_{4}\eta - \frac{a_{3}\sinh a_{1}\eta}{\sinh a_{1}} \] (3.15)

\[ \phi_1 = a_{8}\sinh a_{5}\eta - \frac{a_{7}\sinh a_{2}\eta}{\sinh a_{2}} \] (3.16)

By substituting equations (3.11) to (3.16) into equations (3.1) to (3.3), we obtained solutions for the fluid velocity, temperature and concentration and are presented in the following form

\[ \psi(\eta, \tau) = \left[ \lambda a_{13}[1 - \exp(-a_{10}\eta)] + a_{12}\theta_0 + \frac{a_{16}\sinh a_{10}\eta}{\sinh a_{1}} + \frac{a_{11}\sinh a_{1}\eta}{\sinh a_{1}} \right] + \right. \]

\[ \epsilon \exp(i\omega\tau) \left( \frac{a_{20}\sinh a_{17}\eta}{\sinh a_{17}} + \frac{a_{18}\sinh a_{2}\eta}{\sinh a_{2}} - \frac{a_{19}\sinh a_{5}\eta}{\sinh a_{5}} \right) \] (3.17)
\[ \theta(\eta, \tau) = \left[ \frac{\sinh a_1 \eta}{\sinh a_1} \right] + \epsilon \exp(i \omega \tau) \left[ \frac{\sinh a_2 \eta}{\sinh a_2} \right] \]  
(3.18)

\[ \phi(\eta, \tau) = \left[ a_4 \eta - \frac{a_3 \sinh a_1 \eta}{\sinh a_1} \right] + \epsilon \exp(i \omega \tau) \left[ \frac{a_8 \sinh a_5 \eta}{\sinh a_5} - \frac{a_7 \sinh a_2 \eta}{\sinh a_2} \right] \]  
(3.19)

3.1 Skin friction: From the velocity field, the skin friction at the plate can be obtained, which is given in non-dimensional form as

\[ Cf = \left[ \lambda a_{10} a_{13} \exp(-a_{10} \eta) + a_{12} + \frac{a_{10} a_{16} \cosh a_{10} \eta}{\sinh a_{10}} + \frac{a_{11} a_{11} \cosh a_{11} \eta}{\sinh a_{11}} \right] + \epsilon \exp(i \omega \tau) \left[ \frac{a_{17} a_{20} \cosh a_{17} \eta}{\sinh a_{17}} + \frac{a_{2} a_{18} \cosh a_{2} \eta}{\sinh a_{2}} - \frac{a_{5} a_{10} \cosh a_{5} \eta}{\sinh a_{5}} \right] \]  
(3.20)

3.2 Nusselt number: From the temperature field, we obtained the heat transfer coefficient which is given in non-dimensional form as

\[ Nu = - \left[ \frac{a_1 \cosh a_1 \eta}{\sinh a_1} \right] - \epsilon \exp(i \omega \tau) \left[ \frac{a_2 \cosh a_2 \eta}{\sinh a_2} \right] \]

3.3 Sherwood number: From the concentration field, we obtained the mass transfer coefficient which is given in non-dimensional form as

\[ Sh = - \left[ a_4 - \frac{a_1 a_3 \cosh a_1 \eta}{\sinh a_1} \right] - \epsilon \exp(i \omega \tau) \left[ \frac{a_5 a_8 \cosh a_5 \eta}{\sinh a_5} - \frac{a_2 a_7 \cosh a_2 \eta}{\sinh a_2} \right] \]  
(3.21)

4. RESULTS AND DISCUSSION

A series of computations has been carried out for the effects of the following parameters: rotation parameter $K^2$, upper wall motion parameter $\lambda$, Hall current parameter $m$, thermal Grashof number $Gr$, solutal Grashof number $Gm$, magnetic parameter $M$, permeability parameter $K$, Prandtl number $Pr$, heat source parameter $H$, Schmidt number $Sc$ and Soret number $Sr$ on the fluid primary velocity $U$, secondary velocity $W$, temperature $\theta$, concentration $\phi$, skin friction $Cf$, Nusselt number $Nu$ as well as Sherwood number $Sh$. In the present study following default parameter values are adopted for computations: $\tau = \pi/2$, $K^2 = 0.5$, $\lambda = 0.1$, $Gr = 2$, $Gm = 4$, $M = m = K = Pr = H = Sr = \omega = 1$, $Sc = 0.22$ and $\epsilon = 0.5$. Therefore all the graphs and tables are corresponding to these values unless specifically indicated on the appropriate graph or table.

We notice that, from Fig. 2 the magnitude of the primary velocity component $U$ reduces with an increase in rotation parameter $K^2$ whereas the secondary velocity component $W$ enhances with an increase in rotation parameter $K^2$. This implies that rotation tends to retard the primary velocity whereas it has a reverse effect on the secondary velocity which is in agreement with
the characteristics of Coriolis force which tends to suppress the primary flow for inducing the secondary flow.

![Graph showing influence of \(K^2\) on fluid primary velocity and secondary velocity.](image1)

**Figure 2.** Influence of \(K^2\) on fluid primary velocity and secondary velocity.

![Graph showing influence of \(\lambda\) on fluid primary velocity and secondary velocity.](image2)

**Figure 3.** Influence of \(\lambda\) on fluid primary velocity and secondary velocity.

Fig. 3 depicts the influence of upper wall motion parameter \(\lambda\). As \(\lambda\) is increasing, the fluid primary velocity \(U\) increases across the channel with maximum primary velocity at the upper wall and the fluid secondary velocity \(W\) decreases across the channel with minimum secondary velocity at the upper wall. The influence of the Hall current parameter \(m\) on primary velocity \(U\) and secondary velocity \(W\) is as shown in Fig. 4. It is observed from these graphs that the
primary velocity $U$ and secondary velocity $W$ increases with an increase in the Hall current parameter $m$.

![Figure 4](image_url)

**Figure 4.** Influence of $m$ on fluid primary velocity and secondary velocity.

![Figure 5](image_url)

**Figure 5.** Influence of $Gr$ on fluid primary velocity and secondary velocity.

The influence of Grashof numbers for heat and mass transfer are illustrated in Figs. 5 and 6 respectively on the fluid primary velocity $U$ and secondary velocity $W$. The Grashof number $Gr$ for heat transfer signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. Grashof number $Gm$ for mass transfer defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is observed that there
was a rise in the primary velocity $U$ and secondary velocity $W$ due to the enhancement of thermal buoyancy force $Gr$ and concentration buoyancy force $Gm$.

The influence of magnetic parameter $M$ on the fluid primary velocity $U$ and secondary velocity $W$ is shown in the Fig. 7.

It is noticed that, an increase in the magnetic parameter $M$ decreases the fluid primary velocity $U$ and secondary velocity $W$ due to the resistive action of the Lorenz forces. This implies that magnetic field tends to decelerate fluid flow. Fig. 8 demonstrates the influence of permeability parameter $K$ on the fluid primary velocity $U$ and secondary velocity $W$. It is
observed that, the fluid primary velocity $U$ and secondary velocity $W$ increases on increasing the permeability parameter $K$.

Figs. 9 to 11, shows the plot of primary velocity $U$, secondary velocity $W$, temperature $\theta$ and concentration $\phi$ of the flow field against different values of Prandtl number $Pr$ taking other parameters are constant. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is evident from Figs. 9 to 11, fluid primary velocity $U$, secondary velocity $W$ and temperature $\theta$ decreases on increasing the Prandtl number $Pr$ whereas concentration $\phi$ increases on increasing the Prandtl number $Pr$.

**Figure 8.** Influence of $K$ on fluid primary velocity and secondary velocity.

**Figure 9.** Influence of $Pr$ on fluid primary velocity and secondary velocity.
Figs. 12 to 14, demonstrate the plot of fluid primary velocity $U$, secondary velocity $W$, temperature $\theta$ and concentration $\phi$ for a variety of heat source parameter $H$. It is seen in graphs that, the fluid primary velocity $U$, secondary velocity $W$ and temperature $\theta$ decrease on increasing the heat source parameter $H$ whereas concentration $\phi$ increases on increasing the heat source parameter $H$. This implies that heat source parameter reduced the fluid temperature.

The influence of fluid primary velocity $U$, secondary velocity $W$ and concentration $\phi$ in presence of foreign species such as Hydrogen ($Sc = 0.22$), Helium ($Sc = 0.30$), Water vapour ($Sc = 0.60$) and Ammonia ($Sc = 0.78$) is shown in Figs.15 and 16.
Physically, Schmidt number signifies the relative strength of viscosity to chemical molecular diffusivity. It is noticed that, fluid primary velocity $U$, secondary velocity $W$ and concentration $\phi$ increases on increasing the Schmidt number $Sc$. 
Figs. 17 and 18 demonstrate the influence of Soret number $S_r$ on the primary velocity $U$, secondary velocity $W$ and species concentration $\phi$. It is observed that, primary velocity $U$, secondary velocity $W$ and species concentration $\phi$ increases on increasing the Soret number $S_r$. This implies that, Soret number tends to enhance the fluid primary velocity, secondary velocity and species concentration.

From tables 1 to 3, it is clear that the primary skin friction coefficient increases on increasing the upper wall motion parameter $\lambda$ at both cold and heated plates. The primary skin friction
Figure 17. Influence of $Sr$ on fluid primary velocity and secondary velocity.

Figure 18. Influence of $Sr$ on concentration.

coefficient decreases at the cold wall and increases at the heated wall on increasing the rotation parameter $K^2$, magnetic parameter $M$, Prandtl number $Pr$ and heat source parameter $H$ whereas it is increases at the cold wall and decreases at the heated wall on increasing the Hall current parameter $m$, thermal Grashof number $Gr$, solutal Grashof number $Gm$, permeability parameter $K$, Schmidt number $Sc$ and Soret number $Sr$. The secondary skin friction coefficient decreases at the cold wall and increases at the heated wall on increasing the upper wall motion parameter $\lambda$, magnetic parameter $M$, Prandtl number $Pr$ and heat source parameter $H$ whereas it is increases at the cold wall and decreases at the heated wall on increasing the
rotation parameter \( K^2 \), Hall current parameter \( m \), thermal Grashof number \( Gr \), solutal Grashof number \( Gm \), permeability parameter \( K \), Schmidt number \( Sc \) and Soret number \( Sr \).

From table 4, it is clear that the heat transfer coefficient \( Nu \) increases at the cold wall, decreases at the heated wall on increasing the Prandtl number \( Pr \), heat source parameter \( H \).

From table 5, it is clear that the mass transfer coefficient \( Sh \) decreases at the cold wall and increases at the heated wall on increasing the Prandtl number \( Pr \), heat source parameter \( H \), Schmidt number \( Sc \) and Soret number \( Sr \).

### Table 1. Influence of \( K^2 \), \( m \), \( \lambda \) and \( M \) on the skin friction coefficient.

<table>
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<th>( K^2 )</th>
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<th>( \lambda )</th>
<th>( M )</th>
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TABLE 2. Influence of $K$, $Gr$, $Gm$ and $Pr$ on the skin friction coefficient.

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TABLE 3. Influence of $H$, $Sc$ and $Sr$ on the skin friction coefficient.

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Table 5. Influence of $H$, $Sc$, $Sr$ and $Pr$ on the mass transfer coefficient.

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5. Conclusions

In this paper we have studied analytically the influence of hall current and heat source on unsteady MHD flow of a rotating fluid in a parallel porous plate channel. From the present investigation the following conclusions can be drawn:

- Primary velocity decreases and secondary velocity increases on increasing the rotation parameter. The primary skin friction coefficient decreases at the cold wall and increases at the heated wall whereas the secondary skin friction coefficient increases at the cold wall and decreases at the heated wall with an increase in the rotation parameter.
- Primary velocity increases and secondary velocity decreases on increasing the upper wall motion parameter. The primary skin friction coefficient increases at both cold and heated walls whereas the secondary skin friction coefficient decreases at the cold wall and increases at the heated wall with an increase in the upper wall motion parameter.
- Both the primary and secondary velocity profiles are increases on increasing the Hall current parameter and Soret number. Primary and secondary skin friction coefficients are increases at the cold wall and decreases at the heated wall on increasing the Hall current parameter and Soret number.
- The primary and secondary velocity profiles are decreases on increasing the heat source parameter. Primary and secondary skin friction coefficients are decreases at the cold wall and increases at the heated wall on increasing the heat source parameter.

Acknowledgments

The authors are extremely thankful to the learned referee for his valuable suggestions and comments towards the improvement of the paper.

Nomenclature

- $a$ – distance between two parallel plates
- $B_0$ – uniform magnetic field
- $C$ – species concentration
- $C_f$ – skin-friction coefficient
- $C_1$ – species concentration at the heated wall
- $C_0$ – species concentration at the cold wall
- $c_p$ – specific heat at constant pressure
- $K_1$ – dimensional permeability parameter
- $D_m$ – chemical molecular diffusivity
- $Sr$ – Soret number
- $p$ – fluid pressure
- $Gm$ – Solutal Grashof number
- $Gr$ – thermal Grashof number
- $g$ – acceleration due to gravity
- $H$ – non-dimensional heat source parameter
- $j_w$ – mass flux
- $k_T$ – thermal conductivity of the fluid
- $K^2$ – non dimensional rotation parameter
- $m$ – Hall current parameter
- $M$ – Magnetic parameter
- $Nu$ – Nusselt number
- $n$ – dimensional frequency of oscillation
- $Pr$ – Prandtl number
- $Q_0$ – dimensional heat source parameter
- $q_w$ – heat flux
- $Sc$ – Schmidt number
- $Sh$ – Sherwood number
- $T$ – fluid temperature
- $T_m$ – mean temperature of the fluid
INFLUENCE OF HALL CURRENT AND HEAT SOURCE ON MHD FLOW OF A Rotating Fluid

$T_1$ – fluid temperature at the heated wall  
$T_0$ – fluid temperature at the cold wall  
$t$ – dimensional time  
$U$ – primary velocity  
$W$ – secondary velocity  
u – fluid velocity in $x$ – direction  
v – fluid velocity in $y$ – direction  
w – fluid velocity in $z$ – direction

Greek Symbols

$c$ – coefficient expansion for species concentration  
$\beta_\text{e}$ – coefficient expansion for species concentration  
$\beta_T$ – coefficient of thermal expansion  
$\nu$ – kinematic coefficient of viscosity  
$c_T$ – A scaled coordinate  
$\phi$ – A scaled frequency  
$\phi$ – A scaled temperature  
$\rho$ – fluid density  
$\Omega$ – dimensional rotation parameter  
$\Omega_T$ – A scaled temperature  
$\sigma$ – electrical conductivity  
$\Psi$ – compact velocity  
$\psi$ – compact velocity  
$\tau$ – non dimensional time  
$\tau_w$ – shear stress  
$\tau_{\text{in}}$ – internal heat generation or absorption

APPENDIX

$$a_1 = \sqrt{Pr H}, a_2 = \sqrt{Pr (H + i \omega)}, a_3 = Sc Sr, a_4 = 1 + a_3, a_5 = \sqrt{Sc i \omega},$$

$$a_6 = \frac{a_2^2}{a_2 - a_5^2}, a_7 = a_3 a_6, a_8 = 1 + a_7, a_9 = \frac{M}{1 + im} - 2iK^2, a_{10} = \sqrt{a_9 + \frac{1}{K}},$$

$$a_{11} = \frac{Gma_3 - Gr}{a_{11}^2 - a_{10}^2}, a_{12} = \frac{Gma_4}{a_{10}^2}, a_{13} = \frac{a_9}{a_{10}^2}, a_{14} = \lambda = (a_{11} + a_{12}),$$

$$a_{15} = a_{13} [1 - exp(-a_{10})], a_{16} = a_{14} - a_{15}, a_{17} = \sqrt{a_9 + \frac{1}{K} + i \omega}, a_{18} = \frac{Gma_7 - Gr}{a_2^2 - a_{17}^2},$$

$$a_{19} = \frac{Gma_8}{a_5^2 - a_{17}^2}, a_{20} = a_{19} - a_{18}.$$

REFERENCES

