FINITE DIFFERENCE METHOD FOR A CONSERVATIVE ALLEN–CAHN EQUATION ON NON-FLAT SURFACES

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ABSTRACT

We present an efficient numerical scheme for the conservative Allen–Cahn (CAC) equation on various surfaces embedded in a narrow band domain in the three-dimensional space. We apply a quasi-Neumann boundary condition on the narrow band domain boundary using the closest point method. This boundary treatment allows us to use the standard Cartesian Laplacian operator instead of the Laplace–Beltrami operator. We apply a hybrid operator splitting method for solving the CAC equation. First, we use an explicit Euler method to solve the diffusion term. Second, we solve the nonlinear term by using a closed form solution. Third, we apply a space-time-dependent Lagrange multiplier to conserve the total quantity. The overall scheme is explicit in time and does not need iterative steps; therefore, it is fast. A series of numerical experiments demonstrate the accuracy and efficiency of the proposed hybrid scheme.

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