Inverse Problem in Quantitative Susceptibility Mapping

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ABSTRACT

The inverse problem of QSM is to recover the susceptibility distribution of the human body from the measured local field that is expressed by the convolution of the susceptibility distribution with the magnetic field generated by a unit dipole. However, it is ill-posed due to the presence of zeros at a cone in the Fourier representation of the unit dipole kernel. Due to this ill-posedness, the reconstructed image contains severe streaking artifacts. In this work, we will provide mathematical analysis for the inverse problem of QSM, including the existence and the uniqueness of the solution and the structure of the streaking artifacts for the first time.

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