Review on Supra-convergence of Shortley-Weller method for Poisson equation

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ABSTRACT

The Poisson equation is of primal importance in many physical problems, especially in fluid flows with incompressible condition. And the Shortley-Weller method [2] is a basic finite difference method for solving the Poisson equation with Dirichlet boundary condition. The second order convergence of its solution has been long known but it is rather recent to pay attention to its gradient. Especially in its application to fluid flows, the gradient plays a physical role rather than the solution itself. Since 1938, it was numerically reported that both of the numerical solution and its gradient are second order accurate. We call this increase of the order of accuracy, supra-convergence. At the annual meeting in 2014, we introduced the analysis for supra-convergence of Shortley-Weller method which was not a simple copy but served a basic foundation to go toward the analysis for other supra-convergences [4]. In this talk, we first review the proof that the convergence order of its numerical solution is the second order: though consistency error is first order accurate at some locations, the convergence order is globally second order. We then discuss a discrete divergence theorem for the Shortly-Weller method and prove that the gradient of the solution is $L^2$-second order accurate in general domains [5]. Usually, the gradient of a second order solution is only first order accurate, but the gradient of the Shortly-Weller solution is second order accurate. Recently, the $L_\infty$ supra-convergence for the Shortley-Weller method was completely proved by L. Weynans [3] and revisited by C. Min, J. Seo and S. Ha [1].

REFERENCES